



The 10th Grade Students' Folding Back Process in Solving Contextual Mathematical Problem

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Abstract. Mathematical contextual problem can be useful for students because it is able to evoke higher student's thinking and good mathematization to apply mathematics in real world. However, many students have been difficult in solving mathematical contextual problems, and some of them even have difficulty in understanding mathematical contextual problems. The folding back process is a key feature of the Pirie-Kieren theory about layers of mathematical understanding. Folding back occurs when students cannot solve a problem at an outer level of understanding directly, so they return to the inner level and reconstruct their understanding using their new knowledge. The purpose of this study is to analyze the folding back process in 10th grade students' mathematical understanding based on the Pirie-Kieren theory in solving mathematical contextual problems. Thus, this study used qualitative approach. The subjects of this study were two students of grade 10th in Jakarta. The data were collected by giving two items of mathematical contextual problem and interviewing with semi-structured interview. The result of this study showed that both of the subjects often folding back to the primitive knowing level and go forward to the next level. This study showed that primitive knowing level was a key factor to solve mathematical contextual problems.

Keywords: Mathematical contextual problem, folding back, Pirie-Kieren, layers of understanding, solving problem

INTRODUCTION ~ Mathematics is very important and closely related to human life, so that it is learned by students at every level school education. Educators need to strive conducting meaningful learning which support students to be able to use mathematics correctly and stimulate mathematization well. Mathematical contextual learning is one of learning approaches which is capable to bridge informal to formal mathematics completely (Gravemeijer & Doorman, 1999). Contextual story problems requires ability to interpret the real problems into mathematical form (mathematization). This process is very important, because students' interpretation may lead to error or misconception in working on contextual story (Widjaja, 2013). Students' ability to

choose the best representation for mathematical ideas is necessary since algorithms depend upon their representation of a contextual problem (Samsuddin & Retnawati, 2018).

Results of studies have shown that students still have difficulties in solving contextual story problems (Jupri & Drijvers, 2016; Johar, Patahuddin, & Widjaja, 2017; Hoogland, Pepin, de Koning, Bakker, & Gravemeijer, 2018). Many students had difficulty in composing a picture or diagram based on story problems (Jupri & Drijvers, 2016). In line with this, students who were given pictures or diagrams problem earned greater score than students who were given story or word problems (Hoogland, Pepin, de Koning, Bakker, & Gravemeijer, 2018).

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When students work on word problems, students' thinking will be more complex than doing routine problems, because they should define and solve problems by applying and connecting their prior knowledge and experience (Littlefield & Rieser, 1993; Johar, Patahuddin, & Widjaja, 2017). Pirie-Kieren theory deals with the growth of understanding presenting one framework which can be used to analyze

the process of students' understanding when solving the contextual story problems. Based on Pirie-Kieren theory, mathematical understanding is viewed as a recursive process. Mathematical understanding consists of several nonlinear and recurrent levels (Pirie & Kieren, 1989; Pirie & Kieren, 1994b; Pirie & Kieren, 1994a). Illustration of the growth of understanding can be seen in Figure 1.

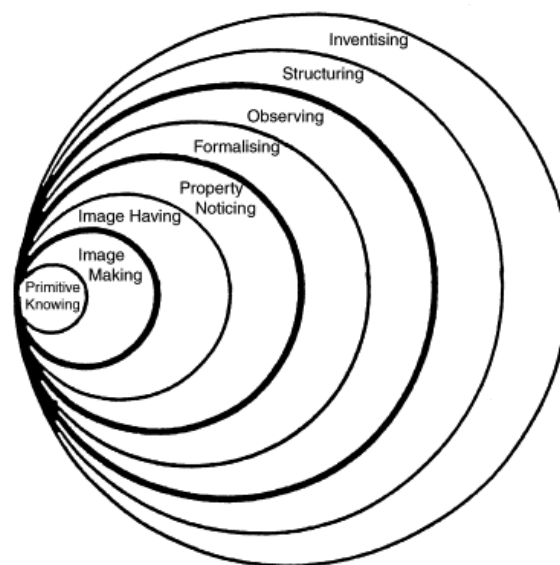


figure 1, Model of Pirie-Kieren theory of the growth of understanding

The level of the Pirie-Kieren theory consists of Primitive Knowing, Image Making, Image Having, Property Noticing, Formalizing, Observing, Structuring, and Inventing. Each level depends on the deeper levels and limited by the outside levels (Pirie & Kieren, 1989). Primitive knowing does not indicate the level of lower-order thinking, but it is the starting point of the growth of understanding (Pirie & Kieren, 1994b). Primitive knowing includes prior knowledge and basic concept that must be understood to resolve the problems.

Image making is the process of composing a picture or schema based on prior knowledge and basic concept as a plan to resolve the problem. At the image having level, students already have a mental picture of the given information, so they do not need to write a picture or schema (Pirie & Kieren, 1994b). Image having is a first level that needs abstraction, and it should be remembered that students should be able to do abstraction in this level. The students must already have a mental picture in this level, so if necessary, students can do the



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image making many level for many times to build their understanding (Pirie & Kieren, 1989). The next level is the property noticing, where students can do abstraction more specifically, which is to understand the difference, relationship, and the combination of image or schema (Pirie & Kieren, 1989).

The next levels include a formal-abstract understanding. In formalizing level, students can solve concrete problems using formal mathematical concepts, such as the definition or theorem. After the students can do formalizing, then students do the observing, that is to re-examine the truth of the answer, and prove it if necessary (Pirie & Kieren, 1994a). Structuring occurs when students understand that there are some related concepts, then verify and establish a system of concepts (Pirie & Kieren, 1989). Last level is inventing, when students created new questions that trigger the discovery of a new concept (Pirie & Kieren, 1989).

Students in the level of inventing have really mastered the previous concepts, which raised the question of "what if" as the opening of a new concept. By the time students begin to understand the new concept, this process will go back over and over. Students's previous understanding would be the primitive knowing for which students will use to learn new concepts.

Each level of the growth of understanding is nonlinear, so the students' understanding does not always move from the inside to the outside. When students want to solve a

problem, but can not finish it directly, they can go back to the lower level of understanding to explore more information or prior knowledge that is needed (Martin, 2008). This process is called folding back. For example, when a student has reached the level of formalizing, but had difficulty solving problem in this level. Students can go back to the primitive knowing level to explore the concepts from prior knowledge that may be related to that problem, or go back to the image making level to make an alternative image or scheme.

Folding back is a crucial characteristic in the Pirie-Kieren theory (Martin, 2008). As well as folding laundry, folding the back also makes the understanding of students become more "thick". The more students perform folding back, the deeper their understanding. (Pirie & Kieren, 1994b). When students do folding back, students do not just repeat a lower level, but their objective is to explore the specific information to build a higher level (Pirie & Kieren, 1989).

Based on several studies, folding back plays an important role when students solving mathematical problems (Martin & Towers, *Folding Back and Growing Mathematical Understanding: A Longitudinal Study of Learning*, 2016; Komatsu, Fujita, Jones, & Sue, 2018; Utomo, Kusmayadi, & Pramodya, 2018; Maboŧja, Chuene, Maoto, & Kibirige, 2018; Susiswo, Subanji, Chandra, Purwanto, & Sudirman, 2019; Nopa, Suryadi, & Hasanah, 2019; Güllilika, Ugurlu, & Yürük, 2015). Folding back helps



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students to explore the information in problem solving (Nopa, Suryadi, & Hasanah, 2019), explore the ideas in finding a general rule (Komatsu, Fujita, Jones, & Sue, 2018), increase geometrical reasoning through the reflection process (Mabotja, Chuene, Maoto, & Kibirige, 2018), and also strengthen the students' basic concepts when solving linear programming problems (Utomo, Kusmayadi, & Pramoedya, 2018). Not only are found in students, folding back is also important to college students of mathematics study program (Susiswo, Subanji, Chandra, Purwanto, & Sudirman, 2019), especially for students who have often solving higher-order thinking problems, so that they become more skeptical and cautious. A longitudinal research study conducted by Martin & Towers for two decades claimed that Pirie-Kieren theory can still be used and developed over time (Martin & Towers, *Folding Back and Growing Mathematical Understanding: A Longitudinal Study of Learning*, 2016). Changes in the education perspectives make it even more powerful theory as the basis of the analysis students' understanding and learning process.

Pirie-Kieren theory is related with the theory of constructivism, which encourages students to construct their own understanding (Pirie & Kieren, 1992). Students got a chance to perform folding back movements to comprehend multiple representations and strengthened their mathematical understanding by revising and reorganizing their previous understandings (Gülkilika, Ugurlu, & Yürük,

2015). Students who have low academic achievements were able to construct concepts by their constant folding backs (Sengul & Argat, 2015). Thus, teachers can use folding back as a pedagogical tool to analyze the process of students' understanding (Martin & Towers, *Folding Back and Growing Mathematical Understanding: A Longitudinal Study of Learning*, 2016).

The impact of folding back is not always positive. Sometimes, even if the student has done folding back, but students do not receive the necessary information (Susiswo, Subanji, Chandra, Purwanto, & Sudirman, 2019). Students have different understandings in solving mathematical problems in accordance with each student's prior knowledge, so that the students' folding back processes are also different (Martin & Towers, *Folding Back and Growing Mathematical Understanding: A Longitudinal Study of Learning*, 2016; Komatsu, Fujita, Jones, & Sue, 2018).

This study aims to analyze the process of students' folding back in solving mathematical contextual problems. When students work on mathematical contextual problems, they should have a good understanding of mathematical concepts to interpret and determine the appropriate solutions. It will be eventful for educators in recognizing and improving students' ability to solve mathematical contextual problems. For example, when the students taking a lot of folding back to the primitive



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knowing level, it means that the teachers need to give an apperception so that they have enough prior knowledge, or when students often folding back to the image making level but can not solve the problem, the teachers need to clarify the intent of the questions, so that the students have enough information to draw up a picture or schema.

METHOD

This study used a qualitative method. The subjects of this study consist of two students of 10th grade in the Jakarta city. Both of the subjects had studied concepts of linear equations system of three variables, and accustomed to solve the routine problems. The data collection method was found by through giving two items of mathematical contextual story problems on linear equations system of three variables. Researcher paid attention to subjects' process to understand the problems and observe the steps performed in the process of resolving the problem. After they completed the problems, the subjects were interviewed by using semi-structured interview to confirm the steps that had been performed understood as well in interpreting and solving the questions. The data obtained from this study were analyzed according to the theory Pirie-Kieren through the qualitative descriptive approach.

RESULTS

Problem number 1 is a mathematical contextual story problem of linear equations system of three variables that is

ended at the observing level. Students do not need to go to the next level, because it just requires necessary to solve the formalizing and observing in checking whether the answer is correct or not. Observing level is optional, since not all students who managed to answer correctly will do the observing. The answer of Subject 1 (S1) contained in Figure 2, while the answer of Subject 2 (S2) contained in Figure 3.

S1 started her work at the image making level, by drawing sketches based on the problems. Then, S1 moved to image having level, by writing and explaining the connection between variables. Researchers confirmed that there is still a linear equation that is wrong, so that S1 went back at the level of image making to see where the problem is. S1 thought her sketches are correct, so that S1 back to the primitive knowing level, by looking at the information provided on the problem 1. S1 managed to find his own mistake after reading the information on the problem 1, so that S1 could reach the image making level with the right answer.


Then, S1 moved to the property noticing level by arranging the linear equations system of three variables and plan her steps to complete the solutions. Researchers found an interesting data, which S1 was about to complete the equation without substituting one value that is known from the task. Further, S1 entered formalizing level, where she completed a system of equations that has been made. S1 create

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a new equation with reference to x variable, because he did not substitute the value of x variable. Later, S1 went back at the image having level to find the value of

x variable. S1 tried to substitute the new equation that she made to the original equation known from the task.

1. Pak Andi adalah seorang pengrajin kayu. Suatu hari, Pak Andi menerima pesanan sebuah pintu yang terdiri dari tiga bagian, yaitu ventilasi di bagian atas, daun pintu di bagian tengah, serta pintu kecil untuk hewan peliharaan di bagian bawah. Kliennya menginginkan pintu hewan peliharaan memiliki tinggi sama dengan tinggi ventilasi ditambah seperlima tinggi daun pintu, serta daun pintu tingginya lima pertujuh tinggi keseluruhan pintu. Jika tinggi ventilasi adalah 20 cm, berapa tinggi keseluruhan pintu yang harus dibuat Pak Andi?



$x = 20$
 $z = x + \frac{1}{5}y \rightarrow z = 20 + \frac{1}{5}y \rightarrow 20 + \frac{1}{5}(100 + 5x)$
 $20 + 20 + x \rightarrow 40 + x$
 $y = \frac{5}{7}(x+y+z)$
 $\rightarrow \frac{5}{7}(x+y+z) = 100 + 5x$
 $\hookrightarrow z = 20 + \frac{1}{5} \cdot \frac{5}{7}(x+y+z)$
 $\frac{5}{7}(x + (100+5x) + (40+x)) = 100 + 5x$
 $z = 20 + \frac{1}{5}x + \frac{1}{5}y + \frac{1}{5}z$
 $\frac{5}{7}(7x + 140) = 100 + 5x$
 $z = 20 + \frac{1}{5}(x+y+z) \cdot 7$
 $= 5x + 100 = 100 + 5x$
 $7z - 140 = x + y + z$
 $6z - 140 = x + y$
 $6(20 + \frac{1}{5}y) - 140 = x + y$
 $120 + \frac{6}{5}y - 140 = x + y$
 $-\frac{20}{5} = -\frac{6}{5}y + y + x$
 $\frac{6}{5}y - y - x = 20$
 $\frac{6-5}{5}y - x = 20$
 $\frac{1}{5}y - x = 20$
 $\frac{1}{5}y = 20 + x$
 $y = 100 + 5x$
 $z = 40 + x$
 $x = x$
 $y = \frac{5}{7}(7x + 140)$
 $y = 5x + 100$
 $x + y + z$
 $\hookrightarrow 20 + 100 + 100 + 40 + 20$
 $= 20 + 200 + 60$
 $= 280 //$

Figure 2. S1's worksheet on contextual problem 1

The first experiment of formalizing level by S1 was failed, because, S1 just found an equation of the same value of x variable, that is $5x + 100 = 100 + 5x$. S1 thought the answer is definitely wrong, so he went back to the primitive knowing level. Researchers gave instructions so that S1 read the questions more carefully. S1 read the

questions while matching the connections between variables, so that S1 perform primitive knowing and image having level continuously. In the end, S1 discovered that the x variable is already known. S1 used that information to move directly towards formalizing level, which in turn S1 can answer correctly.

1. Pak Andi adalah seorang pengrajin kayu. Suatu hari, Pak Andi menerima pesanan sebuah pintu yang terdiri dari tiga bagian, yaitu ventilasi di bagian atas, daun pintu di bagian tengah, serta pintu kecil untuk hewan peliharaan di bagian bawah. Kliennya menginginkan pintu hewan peliharaan memiliki tinggi sama dengan tinggi ventilasi ditambah seperlima tinggi daun pintu, serta daun pintu tingginya lima pertujuh tinggi keseluruhan pintu. Jika tinggi ventilasi adalah 20 cm, berapa tinggi keseluruhan pintu yang harus dibuat Pak Andi?

$V = 20 \text{ cm}$ $a = 20$
 $b = \frac{5}{7} \times (a + b + c)$
 $c = a + \frac{1}{5} b$
 $b = a + b + c$
 $20 + (\frac{5}{7} \times (20 + b + c)) = (20 + \frac{1}{5} b)$
 $b = \frac{5}{7} \times (20 + b) + 20 + \frac{1}{5} b$
 $= \frac{5}{7} \times (40 + b + \frac{1}{5} b) = \frac{5}{7} \times (40 + \frac{6}{5} b)$
 $= \frac{5}{7} \times (\frac{200 + 6}{5} b)$ $b = \frac{200}{7} + \frac{6}{7} b$
 $b - \frac{6}{7} b = \frac{200}{7}$
 $\frac{1}{7} b = \frac{200}{7}$
 $\frac{b}{7} = \frac{200}{7}$
 $b = 200$
 $c = a + \frac{1}{5} (\frac{5}{7} \times \frac{200}{5} b)$
 $= 20 + (\frac{1}{7} \times \frac{200}{5} b)$
 $= 20 + \frac{200}{35} b$
 $c = 20 + \frac{1}{5} \cdot 200$
 $c = 60$
 $a + b + c$
 $20 + 200 + 60$
 280

Figure 3. S2's worksheet on contextual problem 1

S2 began her work on a more basic level, the primitive knowing. S2 first ensure in advance that a given problem consists of linear equations of three variables, and ensure the variables that should be sought. Then, S2 jumped to level image, which was to determine the connections between variables and tried to write the corresponding equations. However, S2 had difficulties in this level. Researchers suggested that S2 should draw or imagine a sketch of the known problem

beforehand. Researchers found an interesting thing, that instead of draw a sketch on paper, S2 saw the original objects located at the scene, and imagine if that was the known image.

After heading to the image making level, S2 went back to the image having level, but still had difficulties. This time, S2 confused about using the fractions that is from known from the problem. S2 went to the primitive knowing level to dig prior knowledges required about the concept of fractions,



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with the help of researchers. Then, S2 back again at the image having level and successfully developed her own linear equations of three variables, and then to the property noticing level by designing a way to solve her linear equations system of three variables. Unlike the S1, S2 substituted all relatable values into the equation as well. S2 focused on finding the value of one variable first.

In formalizing level, S2 again experienced difficulty in solving linear equations system of three variables, because she did not know how to add up $40 + b + \frac{1}{5}b$, so the researchers gave the instructions to add the same variable first. S2 went to the primitive knowing level to dig up information about the sum of fractions, but with the help of researchers. Having overcome these problems, S2 moved towards to formalizing level, and then she answered the problem 1 appropriately.

The contextual story problem number 2 is much more complicated than the contextual story problem number 1. This problem can be explored until the inventing level, when the subject is able to solve linear equations system of three inverse proportion variables. The subjects are required to see that the connections between variables form inverse proportions. S1 and S2 answer to Question 2 are in Figure 4 and Figure 5.

Researchers found an interesting data, which is S1 and S2 are both begin with the same initial steps, although they were given the test at different times. Both were moving from the image having level, then they headed to property noticing level, up to the formalizing level independently. After formalizing level, both of them entered observing level. It turned out that their answer was wrong, because it did not make any sense to them.

Researchers asked both of them to get back to the image having, to explain the connections between the variables they have made. Both still felt there was nothing wrong. Then, researchers asked S1 and S2 back to the primitive knowing level. Researchers gave a brief apperception about direct and inverse proportion.

After getting an apperception, both of subjects returned to construct a new settlement, to the level of formalizing by applying the concept of inverse proportion. Again, S1 had difficulties in determining the variables that must be reversed. S2 also had difficulties, because her understanding about inverse proportion is still shallow. Both returned to the primitive knowing level. Researchers gave an apperception again, this time by explaining how to build the linear equations of inverse proportion variables.

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Bu Rini adalah seorang penjahit dengan dua orang anak bernama Ina dan Ani. Suatu ketika, Bu Rini menerima pesanan satu set kebaya dengan batas waktu pembuatan 2 minggu. Bu Rini dan Ina dapat menyelesaikan pesanan di atas dalam waktu 4 minggu. Jika Bu Rini bekerja bersama Ani, mereka dapat menyelesaikan pesanan dalam waktu 3 minggu. Karena Ina dan Ani belum mahir, mereka berdua membutuhkan waktu 6 minggu untuk menyelesaikan pesanan set kebaya tersebut. Setelah berpikir beberapa saat, Bu Rini mengatakan bahwa waktu yang diberikan tidak cukup. Apakah pernyataan Bu Rini benar? Berapa lama waktu yang diperlukan Bu Rini untuk menyelesaikan satu set kebaya dengan dibantu Ina dan Ani?

Batas waktu = 2 mg
 $x + y = 4 \text{ mg} \Rightarrow x = 4 - y$
 $x + z = 3 \text{ mg} \Rightarrow 4 - y + z = 3 \Rightarrow z = y - 1$
 $y + z = 6 \text{ mg} \Rightarrow y + (y - 1) = 6 \Rightarrow 2y = 7 \Rightarrow y = 3,5$
 $x = 4 - 3,5 = 0,5$
 $z = 3,5 - 1 = 2,5$
 $x + y + z = 0,5 + 3,5 + 2,5 = 6,5 \text{ mg}$

Menentukan: $\frac{1}{x} + \frac{1}{y} = 4 \Rightarrow \frac{1}{0,5} + \frac{1}{3,5} = 2 + \frac{2}{7} = \frac{14}{7} + \frac{2}{7} = \frac{16}{7}$
 $\frac{1}{y} + \frac{1}{z} = 3 \Rightarrow \frac{1}{3,5} + \frac{1}{2,5} = \frac{2}{7} + \frac{2}{5} = \frac{10}{35} + \frac{14}{35} = \frac{24}{35}$
 $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{16}{7} + \frac{24}{35} = \frac{80}{35} + \frac{24}{35} = \frac{104}{35} > \frac{1}{2}$

Kesimpulan, Bu Rini menolak karena waktunya lebih dari 2 minggu.

Figure 4. S1's worksheet on contextual problem 2

S1 immediately understand his mistake. She advanced to the structuring level by establishing a link between the concepts of inverse proportion and linear equations system with three variables. Then, S1 returned to the formalizing level to complete system of equations that has been made, but the level her formalizing is different from the previous ones. The

interesting thing is S1 made an assumption $a = \frac{1}{x}$. Thus, S1 performed formalizing level as usual, without involving fractions. S1 returned to observing level to see if her answer is reasonable. At the end of the solutions, S1 successfully entered the inventing level, for S1 can explain the way of solving linear equations system of three inverse proportion variables.

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2. Bu Rini adalah seorang penjahit dengan dua orang anak bernama Ina dan Ani. Suatu ketika, Bu Rini menerima pesanan satu set kebaya dengan batas waktu pembuatan 2 minggu. Bu Rini dan Ina dapat menyelesaikan pesanan di atas dalam waktu 4 minggu. Jika Bu Rini bekerja bersama Ani, mereka dapat menyelesaikan pesanan dalam waktu 3 minggu. Karena Ina dan Ani belum mahir, mereka berdua membutuhkan waktu 6 minggu untuk menyelesaikan pesanan set kebaya tersebut. Setelah berpikir beberapa saat, Bu Rini mengatakan bahwa waktu yang diberikan tidak cukup. Apakah pernyataan Bu Rini benar? Berapa lama waktu yang diperlukan Bu Rini untuk menyelesaikan satu set kebaya dengan dibantu Ina dan Ani?

Handwritten student work for a contextual problem involving linear equations with fractions. The student uses various methods including substitution, elimination, and matrix operations to solve for the time taken by Bu Rini, Ina, and Ani.

Handwritten notes and equations:

- $x = 4x$
- $y = 3y$
- $z = 6z$
- 7 minggu karena kan Ina sama Ani 6 minggu jadi mungkin 6 masing-masing 3 minggu.
- Rini + Ina = 4 minggu
- $x + 3 = 4$ minggu
- $x = 1$
- jadi total karna bu rini + Ina + Ani $1 + 3 + 3 = 7$ minggu
- per 1 dan 2
- $\frac{4}{x} + \frac{4}{y} = 1$.3
- $\frac{3}{x} + \frac{3}{y} = 1$.4
- $\frac{12}{x} + \frac{12}{y} = 3$
- $\frac{12}{x} + \frac{12}{y} = 4$
- $\frac{12}{x} + \frac{12}{y} = 3$
- $\frac{12}{x} + \frac{12}{y} = 4$
- $\frac{12}{y} - \frac{12}{z} = -1$ (4)
- per 2 dan 3
- $\frac{3}{x} + \frac{3}{z} = 1$.2
- $\frac{6}{y} + \frac{6}{z} = 1$.1
- $\frac{6}{x} + \frac{4}{z} = 2$
- $\frac{6}{y} + \frac{6}{z} = 1$
- $\frac{6}{z} - \frac{6}{y} = 1$ (5)
- Rini : x
- Ina : y
- Ani : z
- Rini + Ani = $3 \left(\frac{1}{x} + \frac{1}{z} \right) = 1$
- $\frac{3}{x} + \frac{3}{z} = 1$ (2)
- Ina + Ani . 6 $\left(\frac{1}{y} + \frac{1}{z} \right) = 1$
- $\frac{6}{y} + \frac{6}{z} = 1$ (3)
- Rini + Ina = $4 \left(\frac{1}{x} + \frac{1}{y} \right) = 1$
- $\frac{4}{x} + \frac{4}{y} = 1$ (1)

Figure 5. S2's worksheet on contextual problem 2

Unlike the S1, after being given apperception for the second time, the S2's answer was still not right. S2 tried to formalizing again, but still could not give meaning to the equations compiled. S2 also had difficulties in solvin linear equations system involving fractions. Finally, S2 gave up on formalizing level.

DISCUSSION

Based on the results, it can be seen that both of the subjects did folding back in an attempt to solve word problems. Illustrations about comparison of S1 and S2 folding back in contextual problem 1 can be seen in Figure 6, while the illustrations about comparison of S1 and S2 folding back in contextual problem 2 can be seen in Figure 7. Based on the two illustrations, it can be

seen that both subject often do folding back to the primitive knowing level.

According to Nopa, Suryadi, and Hasanah (2018), students did folding back to the primitive knowing level due to lack of prior knowledge that is needed in solving problems. In contextual problem 1, S2 performed folding back to the primitive knowing level because she did not understand the concept of fractions, while S1 perform folding back to the primitive knowing level for she was careless. So, there are times when students do folding back to the primitive knowledge level to check the informations given on the problem instead of dig prior knowledge. On contextual problem 2, both subjects did folding back to the primitive knowledge level due to their

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lack of understanding on the inverse proportion concepts. S2 had more difficulties than S1 on this problem, because

S2 is still lacking on the basic concept of the fractions.

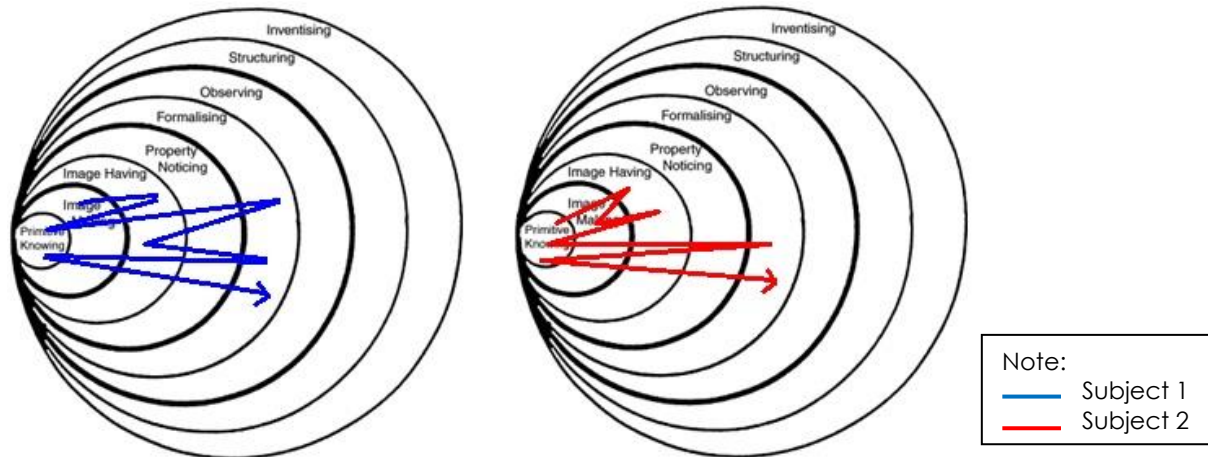


Figure 6. Comparison of S1 and S2 folding back on contextual problem 1

Aside from to the primitive knowing level, both subjects also made several folding back to the image making and image having level. This is in line with the results of Jupri and Drijvers (2016) research which states that students have difficulty in composing a picture or diagram based on the story problems. One interesting result of this research is students do not always describe their image on the answer sheet.

As well as conducting S2 image making level to imagine the original object. Then, students who are able to perform formalizing not necessarily had the right image. When students who had already reached the image having level is still wrong, they must perform folding back if they want to proceed to the formalizing level with the right answer.

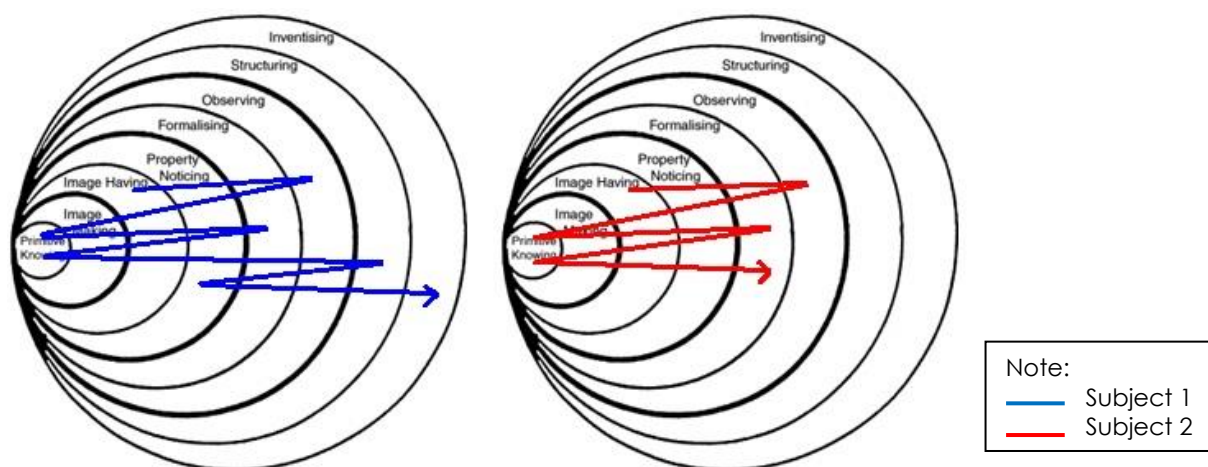


Figure 7. Comparison of S1 and S2 folding back on contextual problem 2



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Both subjects solving number 1 with much different ways, but the result is the same. This is not a problem, because students' approaches in interpreting and solving the contextual story problem can be different according to the respective prior knowledge (Littlefield & Rieser, 1993; Johar, Patahuddin, & Widjaja, 2017). Therefore, the students also have different strategies of folding back (Martin & Towers, 2016; Komatsu, Fujita, Jones, & Sue, 2018).

Problem number 2 requires a higher understanding that is to the level of inventing. At first, both subjects were not aware that the problem involves turning variables to inverse proportion, so that they finish with the concept of linear equations system of three variables as usual. When they reached the observing level, both of them knew that there was one missed concept that should be used. Both subjects do folding back to the primitive knowing level, before advancing to the next level. When students returned to the same level after folding back, the students did not repeat the same level, but the level entered with a deeper knowledge (Pirie & Kieren, 1989).

Folding back is only one of students' attempts to solve problems, so it does not guarantee the student to make the correct answer. It is, for example, in the case of contextual problem 2. S2 had done folding back twice to the primitive knowing level, but still could not get enough information to solve that problem. Susiswo et al. (2019) named this state with pseudo-folding back,

where the students have done folding back, but did not get the necessary information.

There are times when S1 and S2 can not perform folding back independently. Both of them looked confused, but did not realize their mistakes at the previous level. Researchers gave guidance and apperceptions several times due to the lack of prior knowledge of the subject. Martin and Towers (2016) stated that the process of folding back really needed the teacher's role, which is to make the students perform folding back through teacher's intervention, as well as ensuring that the folding back process have been effectively done by the students. Teachers play an important role in the growth of understanding theory Pirie-Kieren, because the teacher is not just responsible to transfer knowledge, but to make sure students are trying to develop their own understanding (Pirie & Kieren, 1994a).

CONCLUSION

There are two main conclusions from this study. First, two subjects were more likely to perform folding back to the primitive knowing level. Sometimes they do not have enough prior knowledge to solve problems. Second, both subjects have not been able to do folding back independently.

REFERENCES

- Gravemeijer, K., & Doorman, M. (1999). Context problems in realistic mathematics education: A calculus course as an example. *Educational*



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- Studies in Mathematics*, 39(1-3), 111-129.
- Gülkilika, H., Ugurlu, H., & Yürük, N. (2015). Examining Students' Mathematical Understanding of Geometric Transformations Using the Pirie-Kieren Model. *Educational Sciences: Theory and Practice*, 15(6), 1531-1548.
- Hoogland, K., Pepin, B., de Koning, J., Bakker, A., & Gravemeijer, K. (2018). Word Problems Versus Image-Rich Problems: An Analysis of the Effects of Task Characteristics on Students' Performance on Contextual Mathematical Problems. *Research in Mathematics Education*, 20(1), 37-52.
- Johar, R., Patahuddin, S., & Widjaja, W. (2017). Linking Pre-Service Teachers' Questioning and Students' Contextual Strategies in Solving Problems: A Case Study in Indonesia and the Netherlands. *The Mathematics Enthusiast*, 14(1), 101-128.
- Jupri, A., & Drijvers, C. (2016). Student Difficulties in Mathematizing Word Problems in Algebra. *Eurasia Journal of Mathematics, Science, and Tehnology Education*, 12(9), 2481-2502.
- Komatsu, K., Fujita, T., Jones, K., & Sue, N. (2018). Explanatory Unification by Proofs in School Mathematics. *For the Learning of Mathematics*, 38(1), 31-37.
- Littlefield, J., & Rieser, J. (1993). Semantic Features of Similarity and Children's Strategies for Identifying Relevant Information in Mathematical Story Problems. *Cognition and Instruction*, 11(2), 133-188.
- Mabotja, S., Chuene, K., Maoto, S., & Kibirige, I. (2018). Tracking Grade 10 Learners' Geometric Reasoning Through Folding Back. *Pythagoras*, 39(1), 1-10.
- Martin, L. (2008). Folding back and the dynamical growth of mathematical understanding: Elaborating the Pirie-Kieren Theory. *The Journal of Mathematical Behavior*, 27(1), 64-85.
- Martin, L., & Towers, J. (2016). Folding Back and Growing Mathematical Understanding: A Longitudinal Study of Learning. *International Journal for the Lesson and Learning Studies*, 5(4), 281-294.
- Nopa, J., Suryadi, D., & Hasanah, A. (2019). The 9th Grade Students' Mathematical Understanding in Problem Solving Based on Pirie-Kieren Theory. *Journal of Physics: Conference Series*, 1157(1), 042122.
- Pirie, S., & Kieren, T. (1989). A Recursive Theory of Mathematical Understanding. *For the Learning of Mathematics*, 9(3), 7-11.



ICEE-2

- Pirie, S., & Kieren, T. (1992). Creating constructivist environments and constructing creative mathematics. *Educational Studies in Mathematics*, 23(5), 505-528.
- Pirie, S., & Kieren, T. (1994a). Beyond Metaphor: Formalising in Mathematical Understanding Within Constructivist Environments. *For the Learning of Mathematics*, 14(1), 39-43.
- Pirie, S., & Kieren, T. (1994b). Growth in Mathematical Understanding: How Can We Characterise It and How Can We Represent It? *Learning Mathematics*, 61-86.
- Samsuddin, A., & Retnawati, H. (2018). Mathematical representation: the roles, challenges and implication on instruction. *Journal of Physics: Conference Series*, 1097(1), 012152.
- Sengul, S., & Argat, A. (2015). The Analysis of Understanding Factorial Concept Processes of 7th Grade Students who have Low Academic Achievements with Pirie Kieren Theory. *The Analysis of Understanding Factorial Concept Processes of 7th Grade Students who have Low Academic Achievements with Pirie Kieren Theory*, 197, 1263-1270.
- Susiswo, Subanji, Chandra, T., Purwanto, & Sudirman. (2019). Folding Back and Pseudo-Folding Back of the Student when Solving the Limit Problem. *Journal of Physics: Conference Series*, 1227(1), 012014.
- Utomo, Kusmayadi, T., & Pramoedy, I. (2018). High Profile Students' Growth of Mathematical Understanding in Solving Linear Programming Problems. *Journal of Physics: Conference Series*, 1008(1), 012070.
- Widjaja, W. (2013). The Use of Contextual Problems to Support Mathematical Learning. *Indonesian Mathematical Society Journal on Mathematics Education*, 4(2), 157-168.