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An Analysis of Mathematical Representation Skills in Solving Problems of Systems of Linear Equations in Two Variables

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Abstract. Mastering mathematics has been a must in order to face today's era of globalization. Mathematical representation is one of the skills that must be possessed and developed by students to solve mathematical problems. Solving mathematical problems requires high level of thinking and understanding of mathematical concepts. Low representation skills, however, may be caused by several factors, one of which is that students are not used to dealing with contextual problems frequently encountered in daily basis. The present study aims to determine the types of mathematical representations exhibited by the students along with their tendency of employing particular mathematical representation(s) in solving the problems regarding the system of linear equations in two variables. The descriptive study with qualitative approach was utilized as the method. A total of 22 grade IX F students of state junior high school 10 Tapung were selected as the subject. This study was conducted to investigate the tendency of students' mathematical representation skills in solving the problems of the system of linear equations in two variables. The results indicate several types of mathematical representations utilized in system of linear equations in two variables, including visual representation of 77%, symbolic representation of 91%, and verbal representation of 27%. In conclusion, the representation skills of the grade IX F students of state junior high school 10 Tapung are still in low category and the students' tendency in solving the problems of system of linear equations in two variables is to use the symbolic representation.

Keywords: Mathematical Representation Skills

INTRODUCTION ~ Mathematics constitutes a basic science and is set as a compulsory subject at every level of education, both primary or secondary ones. Besides being one of the fields of science in education and the development of the other fields, mathematics is a very important subject for students to meet practical needs and solve problems that are frequently encountered in everyday life. This is in line with Sinaga (2016) stating that mathematics is one of the fields of science that can improve someone's ability or skills to think in a logical, rationalist, critical, careful, effective and efficient manner, but to achieve this requires good mathematical understanding and competence. Mathematics mastery is a must in order to face the globalization today.

Likewise, the purposes of mathematics learning are also described in the National Council of Teachers of Mathematics (NCTM, 2000) which includes the ability of problem solving, communication, connecting, reasoning, and representation. It can be concluded that the mathematics learning can help students understand concepts, solve mathematical problems, relate to everyday life, and express mathematical ideas, both orally and in writing, so as to improve their learning outcomes.

Basir & Maharani (2016) state that the curriculum orientation emphasizes on the learning process without forgetting the

goal of mathematics learning namely the achievement of learning outcomes in schools to sharpen the student reasoning in solving problems in daily life, as well as to improve thinking skills in using numbers and symbols in mathematics. One of the abilities or skills that students must possess in learning mathematics based on NCTM is the mathematical representation skills. Nowadays, it is recommended that every student at all educational levels "make and use representations to recognize, record, and communicate mathematical ideas; choose, apply, and translate mathematical representations in order to solve problems; use representations to model and interpret physical, social, and mathematical phenomena" (NCTM, 2000).

Principle and Standard for School Mathematics (in Mustangin, 2015) states that representation is an expression of mathematical thoughts or ideas which are displayed in an attempt to find a solution to the problem being faced. Meanwhile, Hiebert & Carpenter (in Sabirin, 2014) suggest that basically representations can be distinguished in two forms, namely internal and external representations. Thinking about mathematical ideas which are then communicated requires external representations that may be observed in one or more of their forms including verbal state, mathematical equations, images and concrete objects. This is supported by Li (in Zhe, 2012) stating that "the structure of language in mathematical activities includes external communication such as written and oral representation of symbol, word, graphics, and images".

In general, based on the explanation above, it can be concluded that mathematical representation refers to an expression of mathematical ideas that are displayed as models and utilized to find solutions being faced and afterwards become the results of thoughts expressed through images, graphics, words, and mathematical symbols. Moreover, the importance of mathematical representation includes that it can help students develop concepts, understand concepts and express mathematical ideas, and facilitate students to more easily clarify mathematical conditions or problems. In addition, mathematical representation really helps learners understand mathematical concepts in the forms of images, symbols and written words. The use of correct representations by students will help them transform abstract ideas into the more concrete ones as well as form an understanding of mathematical concepts.

The process of developing student mathematical representations are not necessarily to be carried out by providing problems in the form of story problems. Instead, they can be presented in reverse, meaning that the teacher acts to gives symbols, graphs, tables, diagrams, or the other sources, while the students are asked to represent or express them back in the form of stories or real-world problems. It is expected that the representations

expressed through stories can aid to better improve the students' skills of clarifying situation. The different answers given by each student through their story-formed representations might help the teacher see the students' development of representation skills.

When applying a type of mathematical representation to express a typical mathematical idea in solving problems, it is possible that a student has to change that representation into another form. According to Janvier (in Marliyanti, 2016), "the psychological processes involved in going from one mode of representation to another, for example, from an equation to a graph". This means that a change of a type of representation to the other ones denotes translation process.

The mathematical representation skills could be measured by using several mathematical representation indicators. The following indicators were used in this study as referred to Widiati (2012, p. 22):

Table 1. Indicators of mathematical representation skills

No	Aspect of Mathematical Representation Skill	Indicator
1	Visual Representation a. Diagram, graph or table b. Image	• To re-present the data or information obtained from a diagram, graph or table representation • To use visual representations to solve problems • To draw geometric patterns to clarify problems and facilitate resolution
2	Symbolic Representation (Mathematical equation or expression)	• To create mathematical equations or models from the representations given • To make a conjecture of a number pattern • To solve problems by involving mathematical expressions
3	Verbal Representation (written words or texts)	• To create a problem situation based on the data or representation provided • To write down interpretations of a representation and to answer questions either orally or in written • To write down the steps for solving mathematical problems in words • To construct a story that corresponds to the representations presented.

Representation skills are supposed to receive serious attention because if students have access to the representations and ideas that can be displayed, they will obtained a set of tools that can significantly expand their capacity in mathematical thinking (NCTM, 2000, p. 67). However, the fact reveals that, after conducting an interview with a mathematics teacher in one of the junior high schools in Tapung, there were still many students who would answer the questions or problems given merely based on their notes that the teacher had previously provided. Otherwise, if given a question different from the notes in the problem-solving procedure as given by the teacher, many of them would not be able to solve it. This obviously indicates the student low representation skills. Moreover,

low representation skills might be triggered by several factors, one of which is that students are not used to dealing with contextual problems which are, in fact, frequently encountered in daily basis. This condition is in accordance with the data administered by the Research and Development Agency (2015) that the absorption capacity of junior high school students towards mathematics questions in the 2015 National Examination relating to the material of the system of linear equations in two variables is 57.17%. This happened because the students had difficulty determining the value of the variables as well as changing the questions into mathematical models.

Based on the report of The Third International Mathematics and Science Study, it is known that the ability of Indonesian SMP students in the application of: choosing, representing, modeling, implementing, and solving problems is actually low. For instance, when some students were required to present data in the form of tables, bar charts, and circles which are included in the basic competencies of mathematical representation skills, apparently only 14% of Indonesian student participants were able to answer correctly (PPPPTK, 2011). Furthermore, according to the results of Hudiono's research, of all mathematical representation skills, the visual aspect is the weak one (in Widiati, 2012). Based on Widiati (2012), teachers are apparently one of the factors leading to students' lack of mathematical visual representation skills because the teachers provide such representations as tables and pictures to their students only as a complement in delivering their subject matter whereas the teachers rarely pay attention to the representations developed by their students. Meanwhile, the research results conducted by Aryanti et al. (2013) indicate the symbolic representation skill as the one at very low criteria.

According to the interview with one of the teachers in a SMP in Tapung, as well as driven by the relevant previous studies as mentioned above, the researcher of the present study was interested in investigate the mathematical representations in the problem solving of the system of linear equations in two variables. This subject matter was selected as it is considered to continuously apply to further topics. The present study aims to determine the types of mathematical representations exhibited by the students along with their tendency of employing particular mathematical representation(s) in solving the problems of system of linear equations in two variables.

METHOD

The descriptive study with qualitative approach was used as the research method. This is in accordance with the purposes of the study i.e. to figure out the types of mathematical representations presented by the students as well as their tendency of using particular mathematical representation(s) in solving the system of linear equations in two variables problems.

A number of 22 grade IX F students of state junior high school 10 Tapung were chosen as the subject. The data was collected through the measuring technique in the form of mathematical representation test which consists of 3 question items of system of linear equations in two variables subject matter. The study was conducted in the odd semester of the academic year of 2019-2020. In addition to that, interview was also carried out to as the data collection technique. The research instruments were validated by the validators, including a mathematics teacher on the progress of the master study and a junior high school mathematics teachers. Based on the test that have been administered, scores were given and then analyzed to see the students' mathematical representation skills.

The procedures were carried out through 3 stages, i.e.: 1) preparation, 2) implementation, 3) finale.

1) Preparation Stage

The steps taken at the preparation stage consist of: (1) conducting the preliminary study by interviewing the teachers, (2) designing the research, (3) preparing the research instruments in the form of system of linear equations in two variables questions, (4) validating the research instruments, (5) revising the research instruments.

2) Implementation Stage

The steps taken at the implementation stage include: (1) providing the essay-formed test of system of linear equations in two variables, (2) checking the students' work, scoring, and analyzing, (3) selecting several students to be interviewed, (4) processing the data.

3) Final Stage

The steps taken at the final stage are comprised of: (1) analyzing the data gained from the test and interviews, (2) describing the results of data analysis and concluding them as an answers to the research questions, (3) drawing conclusions.

RESULTS

The present study was conducted in the grade IX F of state junior high school 10 Tapung. The subject includes 22 students in total. The class was administered to a test in order to determine the visual, symbolic, and verbal representation skills of the students, and followed by the interviews.

The scores of mathematical representation test are described as follows:

30 60 40 70 40 40 60 30 30 50 60

50 30 30 40 80 100 40 30 30 30 80

Based on the scores gained by the research subject, many of them are still considered in the low category. The mathematical representation of each student in grade IX F can be seen in the following tabel. The results of students' mathematical representation are as follows:

Table 2. Results of Mathematical Representation

No	Item	Indicator		
		1	2	3
1	1	✓		
	2			
	3			
2	1	✓	✓	
	2	✓	✓	
	3			
3	1			
	2		✓	
	3	✓		
4	1	✓	✓	✓
	2	✓		
	3	✓		
5	1		✓	
	2	✓	✓	
	3			
6	1			
	2		✓	
	3		✓	
7	1	✓	✓	
	2			
	3	✓	✓	
8	1			
	2	✓	✓	
	3			
9	1			
	2			
	3	✓		✓
10	1			
	2	✓	✓	✓
	3	✓	✓	
11	1		✓	
	2	✓	✓	
	3			
12	1			
	2	✓	✓	
	3	✓	✓	
13	1		✓	
	2			
	3			
14	1			
	2	✓		
	3			
15	1			
	2	✓	✓	
	3	✓	✓	
16	1	✓	✓	
	2	✓		
	3	✓	✓	
17	1	✓	✓	✓
	2	✓	✓	✓
	3	✓	✓	

18	1	✓		
	2	✓	✓	
	3			
19	1	✓		
	2			
	3			
20	1			
	2		✓	✓
	3			
21	1	✓	✓	
	2			
	3			
22	1	✓		✓
	2	✓	✓	✓
	3	✓	✓	

Where:

- ✓ denotes that the student has met the indicator of mathematical representation.

Based on the table above, it can be concluded that each of the subject is able to make a representation. However, such representation is still categorized as low. From the test questions given by the researcher to the research subject, it can be seen that many of them are able to understand the system of linear equations in two variables symbolically. Furthermore, the table shows those who fulfill the mathematical representation indicators as well. The indicators fulfilled in grade IX of state junior high school 10 Tapung regarding the system of linear equations in two variables materials are elaborated as follows:

17 out of 22 students meet the visual representation indicator of the system of linear equations in two variables, if presented into a percentage, 77% of the students have visual skills while 23% have no visual skills.

20 out of 22 students meet the symbolic representation indicator of the system of linear equations in two variables, if presented into a percentage, 91% of the students have symbolic skills while 23% have no symbolic skills.

6 out of 22 students meet the verbal representation indicator of the system of linear equations in two variables, if presented into a percentage, 27% of the students have verbal skills while 23% have no verbal skills.

15 out of 22 students meet the visual and symbolic representation indicators of the system of linear equations in two variables, if presented into a percentage, 68% of the students have visual and symbolic skills while 32% have no visual and symbolic skills.

5 out of 22 students meet the visual and verbal representation indicators of the system of linear equations in two variables, if presented into a percentage, 23% of the students have visual and verbal skills while 77% have no visual and verbal skills.

5 out of 22 students meet the symbolic and verbal representation indicators of the system of linear equations in two variables, if presented into a percentage, 23% of the students have symbolic and visual skills while 77% have no symbolic and visual skills.

5 out of 22 students meet the visual, symbolic, and verbal representation indicators of the system of linear equations in two variables, if presented into a percentage, 18% of the students have visual, symbolic, and verbal skills while 82% have no visual, symbolic, and verbal skills.

DISCUSSION

Based on the results of this study, it is seen that a variety of mathematical representations are used by students to solve the mathematical problems of system of linear equations in two variables. This include the use of either visual representation only, symbolic representation only, verbal representation only; or visual then symbolic representations or vice versa, visual then verbal representations or vice versa, symbolic then verbal representations or vice versa; or even visual representation followed by symbolic and then verbal representations or vice versa. Nevertheless, only a few students are able to apply various types of representation on one problem. In the process of solving these system of linear equations in two variables problems, some students attempt to interpret them to the mathematical modeling. As stated by NCTM (2000, p.63), representations are used to model and interpret physical, social, and mathematical phenomena. However, many other students have difficulty in modeling the mathematical problems, which in this study is of system of linear equations in two variables. This is in line with the data conveyed by the Research and Development Agency (2015) that the absorption capacity of junior high school students towards mathematics questions in the 2015 National Examination with regard to the system of linear equations in two variables matters is only 57.17% because of the student difficulty in determining the value of the variables and changing the questions to mathematical model. Moreover, in solving the problems given, many students firstly create a table to facilitate the process of making symbols that they understood. This is in accordance with Li (in Zhe, 2012) that "the structure of language in mathematical activities includes external communication such as written and oral representations of symbols, words, graphics, and images".

The results of research conducted by Aryanti et al. (2013) signify that the symbolic representation skill is at very low criteria. This seems to contrast with the present research. In the system of linear equations in two variables materials, the students are more likely to use the symbolic representation. This usage is due to the fact that it is easier and more effective. The researcher find the pattern that the students begin with the visual representation by creating graphs, then

continue to the symbolic representation, or vice versa, as well as those who start from the symbolic representation to the verbal one, or vice versa. Some others use the three indicators of mathematical representation skills, such as from graphs to symbols and to words. Afterwadrs, these results were analyzed by the researcher via interviews with the students, leading to indication that the students tend to use more symbolic representation affected by the teachers' instruction in during the class. The teacher have been oftentimes teaching to employ the symbolic representation on the system of linear equations in two variables subject matter over the other types of representation. This phenomenon is in accordance with Widiati (2012) that teachers apparently become one of the factors causing the students' lack of mathematical visual representation skills since the teachers provide such representations as tables and pictures to their students only as a complement in delivering their subject matter whereas the teachers rarely pay attention to the representations developed by their students. This tendency occurs due to the fact that the symbolic representation makes the students more easily understand the problems especially in the form of story questions.

After all, students are supposed to be capable of using a variety of representations in order to facilitate their learning process. Besides, the teachers should take their part as facilitators for their students' improvement of representation skills. This way, learning can hopefully be more varied and many representations that may occur during the learning process.

CONCLUSION

Based on the results of the study conducted in grade IX F of SMP Negeri 10 Tapung on the material of the system of linear equations in two variables , it is known that in solving the system of linear equations in two variables problems a variety of representations such as symbolic, visual and verbal representations are used. Moreover, to figure out the students' mathematical representation, those who meet the visual representation indicator are as much as 77%, those who meet the symbolic representation indicator are as much as 91%, proving that the tendency of representation usage on the system of linear equations in two variables matter is the symbolic representation. Further, 27% fulfill the verbal representation indicator, 68% fulfill the visual and symbolic representation indicators, 23% fulfill the visual and verbal representation indicators, and finally those who meet all of the three visual, symbolic and verbal indicators are as much as 18%. Therefore, it can be concluded that the representation skills of grade IX students of state junior high school in Tapung are still low because the learning success can only be achieved if if 75% of the whole students meet the target of the learning.

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REFERENCES

Ariyanti, D., Zubaidah, & Nursangaji, A. (2013). Kemampuan representasi matematis menurut tingkat kemampuan siswa pada materi segi empat di SMP. *Jurnal Pendidikan dan Pembelajaran, Portal Garuda*, 2(1), p.4-11.

Badan Penelitian dan Pengembangan. (2015). *Laporan Hasil Ujian Nasional SMP/Mts Tahun Pelajaran 2014/2015*. Jakarta : Puspendik Balitbang Kemendikbud.

Basir, M. A., & Maharani, H. R. (2016, May). Reasoning Ability Students in Mathematics Problems Solving Viewed from Cognitive Style. Prosiding InTHE 2ND INTERNATIONAL SEMINAR ON EDUCATIONAL TECHNOLOGY 2016, p. 99-102.

Maliyanti, D & Amin, S M. (2016). Kemampuan Translasi Antar Representasi Matematika Siswa dalam Memecahkan Masalah Sistem Persamaan Linear Dua Variabel Ditinjau dari Kemampuan Matematika. Jurnal Ilmiah Pendidikan Matematika. 3(5). ISSN: 2301-9085.

Mustangin. (2015). Representasi konsep dan peranannya dalam pembelajaran matematika di sekolah. *Jurnal Pendidikan Matematika*, 1(1), p. 15-12.

NCTM. (2000). *Principles and Standards for School Mathematics*. United States.

Sabirin, Muhammad. (2014). Representasi dalam pembelajaran matematika. 1(2), p. 35.

Sinaga, G. F. M., & Hartoyo, A. (2016). Kemampuan Representasi Matematis Siswa Ditinjau dari Gaya Belajar pada Materi Fungsi Kuadrat di SMA.Jurnal Pendidikan dan Pembelajaran,5(06),1-12.

PPPPTK. (2011). Instrumen hasil belajar matematika SMP : Belajar dari PISA dan TIMSS. Jakarta : PPPPTK Kemendikbud.

Zhe, Liu. (2012). Survey of Primary Students' Mathematical Representation Status and Study on the Teaching Model of Mathematical Representation, 5(1), P. 63-76. South China : Normal University at Guangdong.

Widiati, I. (2012). Mengembangkan kemampuan representasi dan pemecahan masalah matematis siswa SMP dengan penerapan pembelajaran kontekstual. Tesis sps UPI Bandung : tidak diterbitkan.